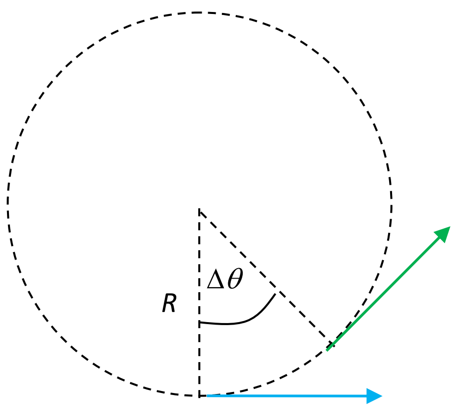
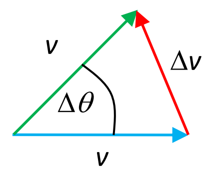
Teacher notes
Topic A

Two simple derivations of centripetal acceleration.

At time zero the velocity vector is shown by the blue arrow. A short time Δt later the velocity vector has changed direction and is shown by the green arrow.





Derivation 1

From $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$ we get $\Delta \vec{v} = \vec{a}\Delta t$ and hence $|\Delta \vec{v}| = |\vec{a}|\Delta t$.

From the figure above $|\Delta \vec{v}| = 2v\sin\frac{\Delta\theta}{2}$; when $\Delta\theta$ is very small $|\Delta \vec{v}| \approx v\Delta\theta$.

But $\Delta\theta = \omega\Delta t$ with $\omega = \frac{v}{R}$ and so $|\Delta \vec{v}| = v\frac{v}{R}\Delta t = \frac{v^2}{R}\Delta t$.

Hence

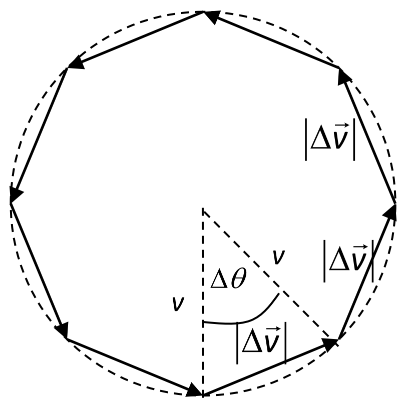
$$|\Delta \vec{v}| = \frac{v^2}{R}\Delta t = |\vec{a}|\Delta t$$

From which follows

$$|\vec{a}| = \frac{v^2}{R}$$

Derivation 2

From $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$ we get $\Delta\vec{v} = \vec{a}\Delta t$ and hence $|\Delta\vec{v}| = |\vec{a}|\Delta t$. In the figure the length of each arrow represents $|\Delta\vec{v}|$. This figure is made of many copies of the small diagram on the right in the figure above.



Summing over all arrows we get $\sum|\Delta\vec{v}| = \sum|\vec{a}|\Delta t$.

When $\Delta\theta$ is very small, the $|\Delta\vec{v}|$ is the same as the length of an arc of a circle of radius *v*. As we sum over the entire circle the sum will give the circumference of this circle which is $\sum|\Delta\vec{v}| = 2\pi v$. And $\sum|\vec{a}|\Delta t = |\vec{a}|T$ where *T* is the period. Thus

$$2\pi v = |\vec{a}|T$$

But $T = \dfrac{2\pi R}{v}$ and so $2\pi v = |\vec{a}|\dfrac{2\pi R}{v}$.

Hence $|\vec{a}| = \dfrac{v^2}{R}$.